

# Assessing Data-Driven Eddy-Parameterizations in an Atlantic Sector Model

David Kamm<sup>1</sup>, Pavel Perezhogin<sup>2</sup>, Julie Deshayes<sup>1,3</sup>, Etienne Meunier<sup>1,4</sup>,  
Alexis Barge<sup>5</sup>

<sup>1</sup>Sorbonne Université-CNRS-IRD-MNHN, LOCEAN Laboratory, Paris, France

<sup>2</sup>Courant Institute of Mathematical Sciences, New York University, New York, NY, USA

<sup>3</sup>Sorbonne Université, Université Paris Cité, CNRS, INRIA, LJLL, EPC ANGE, Paris, France

<sup>4</sup>INRIA, Paris, France

<sup>5</sup>Université Grenoble Alpes, CNRS, IGE, France

## Key Points:

- We implement and test two data-driven eddy parameterizations in the NEMO Ocean model
- The equation-based formulation generalizes better to previously unseen model data than the neural network parameterization
- The equation-based parameterization improves the mean state similarly to an existing backscatter scheme

**Abstract**

Mesoscale eddies are the dominant reservoir of kinetic energy in the ocean and a key component of the Earth’s climate system. Current state-of-the-art climate models can only partially resolve mesoscale processes due to the limited spatial resolution imposed by computational resources, making the accurate parameterization of unresolved kinetic energy transfers crucial. In recent years, a growing number of studies use machine learning tools to build data-driven eddy parameterizations. In this work, we implement two such parameterizations, Zanna and Bolton (2020) (ZB20) and Guillaumin and Zanna (2021) (GZ21), into the NEMO ocean model. We tune and evaluate them against a higher-resolution ground truth, and compare them to a baseline backscatter parameterization in an idealized Atlantic basin configuration. GZ21 is unaware of the model grid, resulting in a systematic bias in the predicted fluxes tied to the local grid spacing. It does not improve the coarse simulation concerning any metrics presented here. ZB20 accurately predicts subgrid fluxes and improves both local kinetic energy spectra and large-scale circulation. It yields an improved mean state comparable to that of the baseline parameterization, consistent with previous studies. We emphasize the importance of carefully designed training data, which takes into account all resolution-dependent model components, such as viscosity and model geometry. Our findings provide guidance for the future development of robust and generalizable data-driven eddy parameterizations.

**Plain Language Summary**

Ocean eddies are important for transporting energy and heat throughout the ocean. They are typically 10 to 100 km wide, too small to be fully captured by current climate models. In this work, we implement two methods using machine learning to represent the effect of unresolved eddies, Zanna and Bolton (2020) (ZB20) and Guillaumin and Zanna (2021) (GZ21). We test them in a computer model of the Atlantic Ocean and compare their performance to a non-machine learning approach and to a high-resolution reference. GZ21 performs poorly, mainly because it does not take into account the grid of the model. ZB20 performed better, improving the energy patterns and ocean circulation of the model similar to the non-machine learning approach. Our results highlight the importance of considering the model grid and other resolution-dependent model components when training machine learning models to represent ocean processes. Our findings can help to improve future developments to better represent ocean eddies.

**1 Introduction**

Simulating Earth’s climate system is strongly constrained by computational capacity, carbon discipline, or both (Acosta et al., 2024). As a result, climate models most often optimize a trade-off between the model’s complexity (the number of processes and climate components explicitly resolved), the spatial resolution of its components (horizontal and vertical), and the temporal duration of integrations (including preliminary experiments for equilibration and calibration of parameters). With the computational resources available today, current climate projections resolve the Oceans with a horizontal grid spacing between 10 km and 100 km (Hewitt et al., 2020). This range spans scales at which mesoscale eddies are not resolved, are partially resolved, or are explicitly resolved. They are the ocean’s primary reservoir of kinetic energy (KE), and not accounting for them appropriately leads to biases in the energy cycle (Ferrari & Wunsch, 2009) and ultimately in the mean state (Juricke et al., 2020). At the largest scales, eddies extract available potential energy (APE) from the mean state, which then cascades down-scale towards the deformation radius. There, baroclinic instabilities transform it into eddy kinetic energy (EKE). An inverse cascade transfers EKE back towards larger scales, thereby energizing the mean flow (Larichev & Held, 1995).

66 In very coarse ocean circulation models (approximately  $1^\circ$  horizontal resolution and  
 67 coarser), even the largest eddies are unresolved, leading to excessive APE, nearly lam-  
 68 inar flow, and erroneous tracer transports. Gent and McWilliams (1990) proposed a pa-  
 69 rameterization for tracer transport driven by baroclinic instabilities, extracting APE from  
 70 the large-scale flow. As computational resources become more affordable, many global  
 71 ocean and climate models enter the so-called "gray zone" of eddy-permitting horizon-  
 72 tal resolution. Depending on the local stratification, latitude, and topography, different  
 73 regions within such models resolve a different range of the eddy spectrum (Hallberg, 2013).  
 74 The unresolved range still needs to be parameterized. At eddy-permitting resolution, it  
 75 is the upscale transfer of EKE from the subgrid scales that is particularly missing, lead-  
 76 ing to an energetically too weak eddy field and mean circulation. Jansen and Held (2014)  
 77 addressed this issue by introducing a prognostic equation for the subgrid KE, which in-  
 78 forms a negative viscosity operator that transfers KE upscale. This kinetic energy backscat-  
 79 ter approach was further developed (Jansen et al., 2015; Klöwer et al., 2018; Juricke et  
 80 al., 2019; Jansen et al., 2019), yet it remains subject to ongoing research.

81 A growing number of studies use machine learning (hereafter ML) tools to build  
 82 data-driven eddy parameterizations (Zanna & Bolton, 2020; Frezat et al., 2021; Guillau-  
 83 min & Zanna, 2021; Guan et al., 2022; Balwada et al., 2025), some of which have already  
 84 been implemented in state-of-the-art ocean models (Zhang et al., 2023; Perezhugin et  
 85 al., 2024). The common approach is to use data from simulations with a high enough  
 86 horizontal resolution to explicitly resolve eddies as "ground truth". The data are filtered  
 87 and coarse-grained to diagnose the subgrid fluxes that are missing at coarse resolution.  
 88 ML models are then trained to predict these fluxes from the coarse-resolution data and  
 89 tested on unseen fields to evaluate their performance. A known issue with data-driven  
 90 eddy parameterizations is that they generalize poorly to new flow regimes (Ross et al.,  
 91 2023; Gultekin et al., 2024), highlighting the importance of an appropriate choice of train-  
 92 ing data (Yan et al., 2024). As with physics-based parameterizations, they need to be  
 93 calibrated to the specific context in which they are employed. This can be challenging,  
 94 especially for black-box parameterizations such as neural networks (hereafter NN), which  
 95 have a vast number of degrees of freedom that are difficult to interpret in physical terms.

96 In this work, we implement two data-driven eddy closures, which target KE backscat-  
 97 ter in the eddy-permitting regime: Zanna and Bolton (2020) (hereafter ZB20) and Guillaumin  
 98 and Zanna (2021) (hereafter GZ21). They take different approaches to the problem. ZB20  
 99 trains a relevance vector machine algorithm to learn equations that best describe the sub-  
 100 grid fluxes from the coarse-grained velocity fields. GZ21 directly predicts the subgrid fluxes  
 101 and their standard deviation using a NN, assuming a Gaussian probability distribution  
 102 for the predicted fluxes. The scientific objective of this paper is to compare the two ap-  
 103 proaches with respect to local energy transfers and their effect on the large-scale ocean  
 104 circulation. As a first step, we introduce the ocean model and the parameterizations to  
 105 be tested (Section 2). We then explain the calibration of parameterizations for the con-  
 106 figuration chosen in this study (section 3) prior to stating the results of the comparison  
 107 for both short- and long-term metrics (section 4). Finally, we discuss all results (section  
 108 5) and conclude (section 6).

## 109 2 Ocean Model and Subgrid Parameterizations

110 We use the Nucleus for European Modelling of the Ocean (NEMO; Madec et al.,  
 111 2023), a well-established modeling framework written in Fortran to simulate ocean dy-  
 112 namics. The DINO configuration (Diabatic Neverworld Ocean; Kamm et al., 2025) was  
 113 developed for NEMO to serve as a testbed for mesoscale eddy parameterizations. It is  
 114 of intermediate complexity, featuring an idealized Atlantic basin geometry, surface forc-  
 115 ing, and a zonally periodic channel representing the Atlantic section of the Southern Ocean.  
 116 As the name suggests, DINO extends the Neverworld2 configuration (Marques et al., 2022)  
 117 by including diabatic processes and an idealized equation of state for temperature and

118 salinity. This allows for the evaluation of eddy parameterizations with respect to their  
 119 role in setting a cross-hemispheric circulation and stratification, in addition to their in-  
 120 teraction with the ocean’s energy cycle. It is thus realistic enough to simulate key met-  
 121 rics of the climate system, such as the Antarctic Circumpolar Current (ACC), Merid-  
 122 ional Overturning Circulation (MOC), and meridional heat transport, but computationally  
 123 inexpensive enough to serve as a hierarchy across a large range of model resolutions.

124 The eddy parameterizations implemented and tested in this study target the eddy-  
 125 permitting regime, where the eddy field is partially resolved. As an example of this regime,  
 126 we use DINO at  $1/4^\circ$ . The unparameterized reference run at this resolution is denoted  
 127 as R4. At  $1/16^\circ$  horizontal resolution (denoted R16), we assume the model to be eddy-  
 128 resolving and treat its solution as ground truth. For details about the configuration and  
 129 the experimental design of R4 and R16, we refer to Kamm et al. (2025). All experiments  
 130 are initialized after a 3000 year spin-up at  $1^\circ$  horizontal resolution to adjust the strat-  
 131 ification to the surface forcing. After interpolation to the respective grids, we run R16  
 132 for 30 years and R4 for 600 years. We use years 20–30 of R16 as a higher-resolution refer-  
 133 ence for assessing short-term metrics, and years 20–30 of R4 for the corresponding un-  
 134 parameterized comparisons. Long-term metrics are assessed using years 550–600 of R4.  
 135 In this section, we introduce the governing equations and the closure problem of the dis-  
 136 cretized model, along with the proposed parameterizations to address it.

## 137 2.1 Momentum equations, coarse-graining and the closure problem

138 We consider the momentum balance for the zonal ( $u$ ) and meridional ( $v$ ) compo-  
 139 nents of the velocity vector field  $\mathbf{u} = \mathbf{u}_h + \mathbf{k}w$  for an incompressible fluid on a rotat-  
 140 ing sphere under the Boussinesq and hydrostatic approximation. The subscript  $[\dots]_h$   
 141 denotes the vector component in the horizontal plane,  $\mathbf{k}$  is the unit vector in the verti-  
 142 cal direction, and  $w$  is the amplitude of velocity in the vertical direction. These equa-  
 143 tions are discretized on grids with different horizontal resolutions for the experiments  
 144 presented in this study. To account for this in the notation, we follow Guillaumin and  
 145 Zanna (2021), denoting fields discretized at coarse and fine resolutions with the super-  
 146 scripts  $\downarrow$  and  $\uparrow$ , respectively. Hence, the momentum equation at coarse resolution (R4)  
 147 reads

$$148 \quad \partial_t \mathbf{u}_h^\downarrow + \left[ (\nabla \times \mathbf{u}^\downarrow) \times \mathbf{u}^\downarrow + \frac{1}{2} \nabla \mathbf{u}^{\downarrow 2} \right]_h + f \mathbf{k} \times \mathbf{u}_h^\downarrow + \frac{1}{\rho_0} \nabla_h p^\downarrow = \mathbf{F}^{\mathbf{u}^\downarrow} + \mathbf{D}^{\mathbf{u}^\downarrow}, \quad (1)$$

149 where  $p$  is the pressure,  $f$  is the Coriolis frequency,  $\rho_0$  is a reference density,  $\mathbf{F}^{\mathbf{u}}$   
 150 represents wind forcing at the surface ( $z = 0$ ) and friction at the bottom ( $z = D$ ). The vis-  
 151 cous terms are collected in  $\mathbf{D}^{\mathbf{u}}$  and will be discussed in detail later. Similarly, the mo-  
 152 mentum equation at high resolution (R16) reads

$$153 \quad \partial_t \mathbf{u}_h^\uparrow + \left[ (\nabla \times \mathbf{u}^\uparrow) \times \mathbf{u}^\uparrow + \frac{1}{2} \nabla \mathbf{u}^{\uparrow 2} \right]_h + f \mathbf{k} \times \mathbf{u}_h^\uparrow + \frac{1}{\rho_0} \nabla_h p^\uparrow = \mathbf{F}^{\mathbf{u}^\uparrow} + \mathbf{D}^{\mathbf{u}^\uparrow}, \quad (2)$$

154 We consider Equation 2 to describe the time evolution of the velocity field  $\mathbf{u}^\uparrow$  at suffi-  
 155 ciently high horizontal resolution to explicitly resolve mesoscale eddies. We treat it as  
 156 our target ground truth and neglect processes that are not resolved at R16, namely the  
 157 submesoscale.

158 To define the closure problem, we introduce a coarse-graining operator  $\overline{(\cdot)}$ , which  
 159 maps fine-grid variables from R16 onto the coarser grid of R4. The coarse-grained R16  
 160 variables are denoted C16 in the following. We perform the coarse-graining by first ap-  
 161 plying a Gaussian spatial filter to suppress small-scale variability, followed by a spatial  
 162 average over the coarse grid cell. To a first approximation, such an operator commutes  
 163 with spatial and temporal derivatives and acts only in the lateral direction. However,  
 164 Aluie (2019) demonstrates that this assumption is not strictly valid for vector fields dis-  
 165 cretized on a sphere. For simplicity, we adopt it nonetheless. By applying the coarse-

166 graining operator to each term of Equation 2, we can formulate the coarse-grained mo-  
 167 mentum equation as

$$168 \quad \partial_t \overline{\mathbf{u}}^\uparrow_h + \left[ (\nabla \times \overline{\mathbf{u}}^\uparrow) \times \overline{\mathbf{u}}^\uparrow + \frac{1}{2} \nabla \overline{\mathbf{u}}^{\uparrow 2} \right]_h + f \mathbf{k} \times \overline{\mathbf{u}}^\uparrow_h + \frac{1}{\rho_0} \nabla_h \overline{p}^\uparrow = \mathbf{F}^{\overline{\mathbf{u}}^\uparrow} + \mathbf{D}^{\overline{\mathbf{u}}^\uparrow} + \mathcal{S} - \mathcal{D}. \quad (3)$$

It takes essentially the same form as Equation 1, with additional subgrid eddy momentum fluxes  $\mathcal{S}$  and  $\mathcal{D}$  arising from the nonlinearity of the advection and viscosity terms. These terms close the coarse-grained momentum equation with respect to the high-resolution solution and are given by

$$\mathcal{S} = \left[ (\nabla \times \overline{\mathbf{u}}^\uparrow) \times \overline{\mathbf{u}}^\uparrow + \frac{1}{2} \nabla \overline{\mathbf{u}}^{\uparrow 2} \right]_h - \overline{\left[ (\nabla \times \mathbf{u}^\uparrow) \times \mathbf{u}^\uparrow + \frac{1}{2} \nabla \mathbf{u}^{\uparrow 2} \right]_h} \quad (4)$$

$$\mathcal{D} = \mathbf{D}^{\overline{\mathbf{u}}^\uparrow} - \overline{\mathbf{D}^{\mathbf{u}^\uparrow}}. \quad (5)$$

169 They reflect the momentum fluxes across the filter scale of the coarse-graining opera-  
 170 tor. The idea of subgrid closures is that by including them in the coarse model, its so-  
 171 lution  $\mathbf{u}^\downarrow$  approaches the coarse-grained ground truth  $\overline{\mathbf{u}}^\uparrow$ . The closure problem is that  
 172 the fluxes depend on the unknown fine-scale velocity field  $\mathbf{u}^\uparrow$  and need to be parame-  
 173 terized for coarse-resolution model simulations.

174 The viscosity terms collected in  $\mathbf{D}^{\mathbf{u}}$  are themselves parameterized. They are de-  
 175 voted to the dissipation of KE cascading toward the grid scale. This is not strictly mo-  
 176 tivated by physical principles but rather serves to ensure the numerical stability of the  
 177 model (Madec et al., 2023). For vertical viscosity, we use the turbulent kinetic energy  
 178 closure (Gaspar et al., 1990), denoted as  $\mathbf{D}_{\text{TKE}}^{\mathbf{u}}$ . Since it does not explicitly depend on  
 179 the horizontal resolution of the grid, the associated subgrid fluxes are close to zero, i.e.,  
 180  $\mathbf{D}_{\text{TKE}}^{\overline{\mathbf{u}}^\uparrow} \approx \mathbf{D}_{\text{TKE}}^{\mathbf{u}^\uparrow}$ . We chose the *biharmonic Smagorinsky* model for horizontal viscos-  
 181 ity (Smagorinsky, 1963; Griffies & Hallberg, 2000), denoted as  $\mathbf{D}_{\text{SMA}}^{\mathbf{u}}$ , with the default  
 182 settings provided in NEMO (see Madec et al., 2023, for details). It is both grid- and flow-  
 183 dependent and highly scale-selective, dissipating only near the smallest resolved scales.  
 184 Consequently, coarse-graining the Smagorinsky term removes most of its associated dis-  
 185 sipation, leaving  $\mathbf{D}_{\text{SMA}}^{\overline{\mathbf{u}}^\uparrow} \gg \mathbf{D}_{\text{SMA}}^{\mathbf{u}^\uparrow}$  and thus  $\mathcal{D} \approx \mathbf{D}_{\text{SMA}}^{\overline{\mathbf{u}}^\uparrow}$ . Substituting into the right-  
 186 hand side of Equation 3 leaves us with

$$187 \quad \partial_t \overline{\mathbf{u}}^\uparrow_h + \dots = \mathbf{F}^{\overline{\mathbf{u}}^\uparrow} + \mathbf{D}_{\text{TKE}}^{\overline{\mathbf{u}}^\uparrow} + \mathcal{S}, \quad (6)$$

188 rendering Equation 3 horizontally inviscid. This implies that our target fields  $\overline{\mathbf{u}}^\uparrow_h$  are not  
 189 a stable solution of the coarse-discretized model (Equation 1). For the sake of numer-  
 190 ical stability, we therefore accept the excessive dissipation introduced by the viscous term  
 191 and set  $\mathcal{D} = 0$  in a-posteriori simulation rollouts. Introducing a subgrid parameter-  
 192 ization  $\tilde{\mathcal{S}}$  in the coarse-discretized model (Equation 1) to account for the fluxes of KE from  
 193 unresolved processes then reads

$$194 \quad \partial_t \mathbf{u}^\downarrow_h + \dots = \mathbf{F}^{\mathbf{u}^\downarrow} + \mathbf{D}_{\text{TKE}}^{\mathbf{u}^\downarrow} + \mathbf{D}_{\text{SMA}}^{\mathbf{u}^\downarrow} + \tilde{\mathcal{S}}, \quad (7)$$

195 When comparing Equation 6 and Equation 7, it becomes clear that to assess parame-  
 196 terizations with respect to the coarse-grained ground truth, the combined parameter-  
 197 ized subgrid fluxes of  $\mathbf{D}_{\text{SMA}}^{\mathbf{u}^\downarrow}$  and  $\tilde{\mathcal{S}}$  need to be considered to approximate the diagnosed  
 198 subgrid fluxes  $\mathcal{S}$ . The KE transfers due to the diagnosed fluxes on the coarse-grained ve-  
 199 locities are then given by

$$200 \quad \mathcal{T} = \overline{\mathbf{u}}^\uparrow \cdot \mathcal{S} \quad (8)$$

201 while those due to the parameterized fluxes on the coarse velocities are given by

$$202 \quad \tilde{\mathcal{T}} = \mathbf{u}^\downarrow \cdot \left[ \mathbf{D}_{\text{SMA}}^{\mathbf{u}^\downarrow} + \tilde{\mathcal{S}} \right] \quad (9)$$

203 In the eddy-permitting regime,  $\mathcal{T}$  typically represents the net upscale kinetic en-  
 204 ergy transfer from unresolved subgrid eddies to the resolved eddy field. However, the trans-  
 205 fer can be either upscale or downscale locally, depending on the instantaneous velocity  
 206 field. The goal of eddy parameterizations in this regime is to predict these fluxes accu-  
 207 rately from coarse-resolution data. The data-driven approaches tested here aim to ad-  
 208 dress this by learning to infer the fluxes using diagnosed fields from high-resolution sim-  
 209 ulations as training data. In the following sections, we introduce the methodology and  
 210 implementation, alongside a physics-based kinetic energy backscatter parameterization  
 211 for reference.

## 212 **2.2 Kinetic energy backscatter parameterization (KEB20)**

213 We employ the kinetic energy backscatter parameterization as implemented and  
 214 tested in NEMO in Perezhgin (2020). It aims to represent the inverse cascade of KE  
 215 from the subgrid-scale, which is missing at eddy-permitting resolution. The kinetic en-  
 216 ergy dissipated by the biharmonic viscosity term is diagnosed and re-injected through  
 217 a negative Laplacian viscosity operator.

$$218 \quad \tilde{\mathcal{S}}_{KEB} = \nabla_h \left( \nu \nabla_h \mathbf{u}_h^\downarrow \right) \quad (10)$$

219 The backscatter viscosity coefficient  $\nu(t, z, y, x) \leq 0$  is computed locally from the sub-  
 220 grid kinetic energy  $e(t, z, y, x)$  as

$$221 \quad \nu = -c_{\text{back}} \Delta^\downarrow \sqrt{\max(e, 0)}, \quad (11)$$

222 where  $c_{\text{back}} = 0.4 \sqrt{2}$ , following Perezhgin (2020), and  $\Delta^\downarrow$  denotes the respective grid  
 223 spacing at coarse resolution. The subgrid kinetic energy follows from an additional prog-  
 224 nostic equation, as first proposed in Jansen and Held (2014) and further developed in  
 225 Jansen et al. (2015), Jansen et al. (2019), and Juricke et al. (2019). Here, it is defined  
 226 as

$$227 \quad \partial_t e = \gamma_{KEB} \dot{E}_{\text{diss}} + \dot{E}_{\text{back}} + \nu_e \nabla_h^2 e, \quad (12)$$

228 where  $\dot{E}_{\text{diss}} \geq 0$  is the rate of energy dissipation by the biharmonic Smagorinsky op-  
 229 erator,  $\dot{E}_{\text{back}} \leq 0$  is the rate of energy backscatter by the negative Laplacian viscos-  
 230 ity operator Equation 10, and the last term corresponds to the diffusion of subgrid ki-  
 231 netic energy with a diffusivity  $\nu_e = 300 \text{ m}^2 \text{ s}^{-1}$ .  $\gamma_{KEB} \approx 1$  is a non-dimensional coef-  
 232 ficient that controls the strength of the backscatter parameterization (Kl ower et al., 2018;  
 233 Juricke et al., 2019).

## 234 **2.3 Zanna-Bolton Parameterization (ZB20)**

235 Zanna and Bolton (2020) write the subgrid eddy fluxes as the divergence of the stress  
 236 tensor  $\mathbf{T}$

$$237 \quad \mathcal{S} = \nabla_h \cdot \mathbf{T}. \quad (13)$$

238 The authors used an equation discovery algorithm to approximate the components of  
 239 the stress tensor in terms of the coarse velocity fields. As basis functions, they chose rel-  
 240 ative vorticity  $\zeta = \partial_x v^\downarrow - \partial_y u^\downarrow$ , horizontal divergence  $\delta = \partial_x u^\downarrow + \partial_y v^\downarrow$ , shearing de-  
 241 formation  $\varepsilon_s = \partial_y u^\downarrow + \partial_x v^\downarrow$ , and stretching deformation  $\varepsilon_n = \partial_x u^\downarrow - \partial_y v^\downarrow$ . The diag-  
 242 nosed subgrid fluxes  $\mathcal{S}$  from the same coarse-graining approach as presented here served  
 243 as the ground truth. The authors used high-resolution data generated from an idealized  
 244 double-gyre configuration as training data, similar to the configuration used in Cooper  
 245 and Zanna (2015). They arrived at the following equation to approximate the stress ten-  
 246 sor:

$$247 \quad \tilde{\mathbf{T}} = \kappa \left[ \begin{pmatrix} -\zeta \varepsilon_s & \zeta \varepsilon_n \\ \zeta \varepsilon_n & \zeta \varepsilon_s \end{pmatrix} + \frac{1}{2} (\zeta^2 + \varepsilon_s^2 + \varepsilon_n^2) \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \right] \quad (14)$$

248 The discovered equation resembles a special case of the nonlinear gradient model, de-  
 249 rived in earlier LES studies through a Taylor-series expansion of the subgrid fluxes (e.g.,

Leonard, 1975; Meneveau & Katz, 2000). We refer to Jakhar et al. (2024) for a detailed discussion on the similarities between the equation discovery and Taylor-series methods. When tested *a posteriori* in an idealized eddy-permitting ocean model, Perezhugin et al. (2024) found that the ZB20 parameterization underestimates backscatter and addressed this issue by applying spatial filtering schemes to suppress noise close to the grid scale and enhance the KE backscatter towards larger scales. The ZB20 parameterization evaluated here corresponds to "ZB-smooth" in their paper, where a Gaussian-weighted spatial low-pass filter is convoluted  $N$  times with each component of the stress tensor:

$$\tilde{\mathcal{S}}_{ZB} = \nabla_h \cdot \left( \mathcal{G}^N * \tilde{\mathbf{T}} \right), \quad (15)$$

where  $\mathcal{G}$  is defined on a  $3 \times 3$  horizontal stencil, given by

$$\mathcal{G} = \frac{1}{16} \begin{pmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{pmatrix}. \quad (16)$$

They found  $N = 4$  to be optimal in the eddy-permitting regime; therefore, we will adopt this value. In Zanna and Bolton (2020), the value for  $\kappa$  is approximated as the averaged regression coefficients from the equation discovery algorithm. Following Perezhugin et al. (2024) and previous nonlinear gradient model studies (Meneveau & Katz, 2000; Chen et al., 2003), we associate the coefficient with the local grid scale  $\Delta^\downarrow$ :

$$\kappa = -\gamma_{ZB} \Delta^{\downarrow 2} \quad (17)$$

This leaves us with a non-dimensional tuning coefficient  $\gamma_{ZB} \geq 0$  to control the magnitude of the momentum fluxes, similar to  $\gamma_{KEB}$  in the previous section.

## 2.4 Guillaumin-Zanna parameterization (GZ21)

Guillaumin and Zanna (2021) takes a stochastic approach to predict both the mean and standard deviation of a Gaussian distributed subgrid momentum forcing. GZ21 is a *convolutional neural network* (CNN) trained on high-resolution surface data from the CM2.6 coupled climate model (Delworth et al., 2012). The coarse-graining procedure for diagnosing the ground-truth subgrid fluxes is similar to the one presented here, but it maps from  $0.1^\circ$  to  $0.4^\circ$  horizontal resolution, targeting slightly coarser models than the test configuration used in this study. The CNN uses the coarse-resolution horizontal velocity field as input to infer the mean and standard deviation of a Gaussian distribution representing the instantaneous subgrid fluxes. The subgrid forcing entering the right-hand side of the momentum equations is then sampled as

$$\tilde{\mathcal{S}}_{GZ} = \gamma_{GZ} \left[ \text{CNN}^{(\text{mean})}(\mathbf{u}_h^\downarrow) + \mathcal{W} \text{CNN}^{(\text{std})}(\mathbf{u}_h^\downarrow) \right], \quad (18)$$

where  $\mathcal{W}$  is a spatial random field sampled from uncorrelated standard normal distributions (i.e., white noise), and  $\gamma_{GZ} \geq 0$  is again a non-dimensional tuning coefficient. The calibration procedure for the tuning coefficients is described in Section 3.

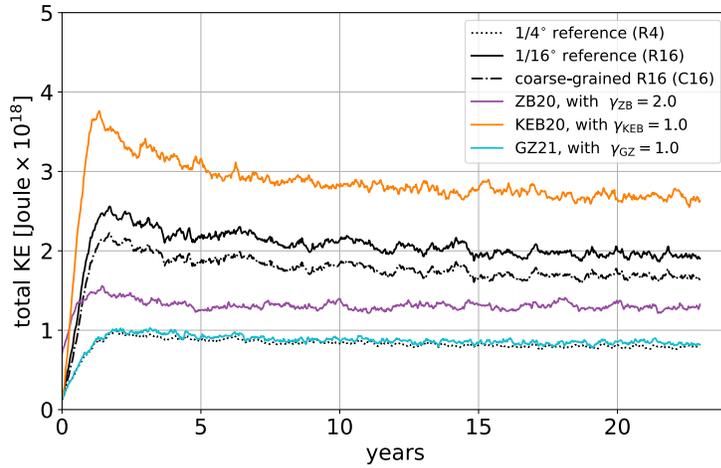
In the original publication, the parameterization is tested *a posteriori* in a barotropic double gyre configuration (Klöwer et al., 2018). In a later study (Zhang et al., 2023), the parameterization was introduced into the Modular Ocean Model version 6 (MOM6 Adcroft et al., 2019) and tested in a baroclinic two-layer model. They found that the model lacks generalization across different depths and horizontal resolutions, as it was trained only on surface data and at a single coarse resolution.

The CNN consists of eight convolutional layers, with approximately a quarter of a million weights learned during its training phase using the Python package PyTorch. To couple the CNN from Python to the NEMO model, we employ the *EOPHIS* package (Barge, Le Sommer, & IGE-MEOM, 2024; Barge, Meunier, & Kamm, 2024). It enables the integration of arbitrary Python code into NEMO via its native coupler, OASIS (Craig et al., 2017). At each time step, the velocity fields are sent to parallel Python

296 processes, where the subgrid forcing is inferred according to Equation 18, sent back to  
 297 the NEMO processes, and finally applied to update the right-hand side of the time-stepping  
 298 scheme.

### 299 3 Tuning strategy

300 The parameterizations presented above include parameters that remain to be cho-  
 301 sen (namely  $\gamma_{KEB}$ ,  $\gamma_{ZB}$ , and  $\gamma_{GZ}$ ), a phase referred to as "tuning" in climate model-  
 302 ing (Hourdin et al., 2017). They were developed for a specific ocean model and tested  
 303 in a specific configuration. The chosen parameters need to be revisited, as we are using  
 304 another ocean model in yet another configuration. We illustrate this by highlighting the  
 305 flaws in using the initially chosen parameters for each parameterization after implemen-  
 306 tation, as part of an initial evaluation of the domain-integrated KE (see Figure 1), prior  
 to explaining our tuning approach.



**Figure 1.** Kinetic energy integrated over the entire domain for R4 (black dotted), R16 (black solid), C16 (black dash-dotted) and the untuned ZB20 (purple), GZ21 (blue) and KEB20 (orange) experiment.

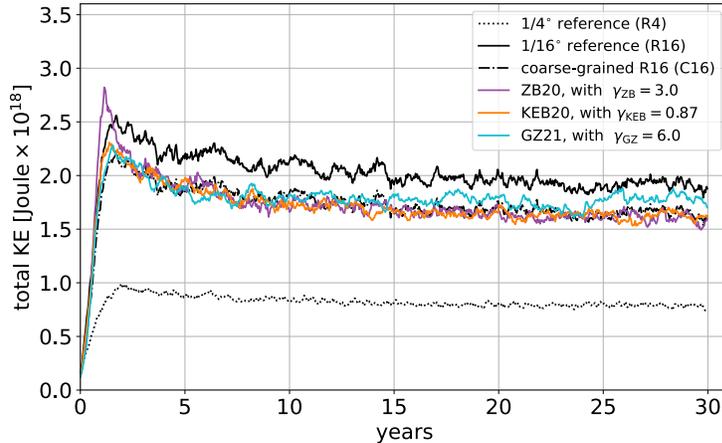
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308 As shown by Kamm et al. (2025), the subgrid fluxes resolved in C16 lead to an up-  
 309 scale transfer of KE from the subgrid to resolved scales. This results in more than twice  
 310 the total KE when compared to R4: a target we want to reach by parameterizing these  
 311 fluxes. Using  $\gamma_{KEB} = 1.0$ , KEB20 overestimates the integrated backscatter by almost  
 312 a factor of two, while GZ21 with  $\gamma_{GZ} = 1.0$  barely energizes the flow at all (Figure 1).  
 313 ZB20 with  $\gamma_{ZB} = 2.0$  is close to the target, which can be explained by the fact that ZB20,  
 314 as implemented here following Perezhogin et al. (2024), was tuned in NW2, a very simi-  
 315 lar configuration to DINO. For a meaningful comparison of the parameterizations, it is  
 316 essential to calibrate them in a common context and with respect to common evalua-  
 317 tion metrics.

318 The most straightforward approach is to directly compare the diagnosed subgrid  
 319 momentum fluxes  $\mathcal{S}$  (Equation 4) to the instantaneously predicted fluxes from each pa-  
 320 rameterization, namely  $\mathcal{S}_{KEB}$  (Equation 10),  $\mathcal{S}_{ZB}$  (Equation 15), and  $\mathcal{S}_{GZ}$  (Equation 18).  
 321 This a priori approach is intuitive for data-driven parameterizations, as they were trained  
 322 on instantaneous fluxes similarly. However, it also comes with similar challenges: it im-  
 323 plicitly assumes that the coarse-grained high-resolution solution of Equation 6,  $\mathbf{u}^\uparrow$ , be-  
 324 haves as the coarse-discretized low-resolution fields  $\mathbf{u}^\downarrow$  of Equation 1. As discussed in

325 Section 2.1, this is not strictly the case, partly due to the additional viscous dissipation  
 326 from the Smagorinsky term required at coarse resolution.

327 Instead, we tune each parameterization a posteriori through repeated simulation  
 328 rollouts. We use the domain-integrated KE to optimize the scalar coefficients  $\gamma_{\text{KEB}}$ ,  $\gamma_{\text{ZB}}$ ,  
 329 and  $\gamma_{\text{KEB}}$  for a minimal root-mean-square error (RMSE) over the last four years of 23-  
 year-long tuning rollouts. We found the optimal values with respect to domain-integrated



**Figure 2.** Kinetic energy integrated over the entire domain for R4 (black dotted), R16 (black solid), C16 (black dash-dotted) and the tuned ZB20 (purple), GZ21 (blue) and KEB20 (orange) experiment.

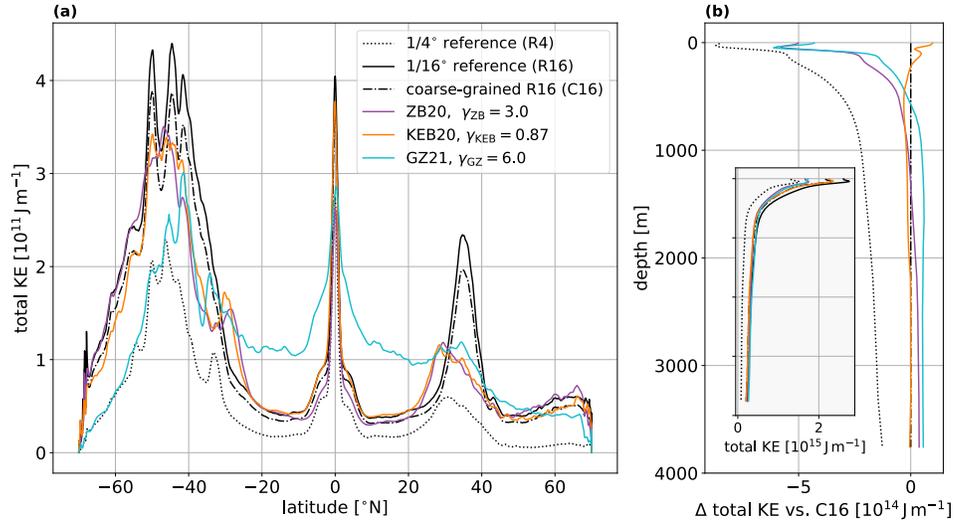
330 KE to be  $\gamma_{\text{KEB}} = 0.87$ ,  $\gamma_{\text{ZB}} = 3.0$ , and  $\gamma_{\text{GZ}} = 6.0$ . These experiments were then con-  
 331 tinued for a total simulation length of 30 years for comparison (see Figure 2).  
 332

## 333 4 Results

334 Since the domain-integrated KE was used for tuning, it cannot serve as an eval-  
 335 uation metric for comparison. Additionally, it does not reveal whether the flow is ener-  
 336 gized in the correct location and at the appropriate scales. In the following, we focus on  
 337 the spatial and spectral patterns of kinetic energy and its associated transfers. Then, we  
 338 compare their impact on long-term metrics of the ocean circulation, which are crucial  
 339 for the climate system.

### 340 4.1 Short-term metrics

341 We decompose the total domain-integrated KE into meridional and vertical pro-  
 342 files (panels **(a)** and **(b)** of Figure 3, respectively). The high-resolution reference exper-  
 343 iment (R16, black solid line) exhibits consistently higher KE across all latitudes and depths  
 344 compared to the low-resolution reference experiment (R4, black dotted line). These dif-  
 345 ferences are most pronounced in regions where the flow is most energetic, namely, verti-  
 346 cally towards the surface (panel **(b)**) and meridionally in the Southern Ocean, the Equa-  
 347 torial region, and close to the Gulf Stream separation (panel **(a)**). After coarse-graining  
 348 the high-resolution reference experiment, most of this additional kinetic energy is retained  
 349 (C16, black dashed line). This demonstrates how energy is not only introduced at the  
 350 expanded spectrum of smaller, resolved scales but also transferred to larger scales cap-  
 351 tured at coarse resolution throughout the domain.



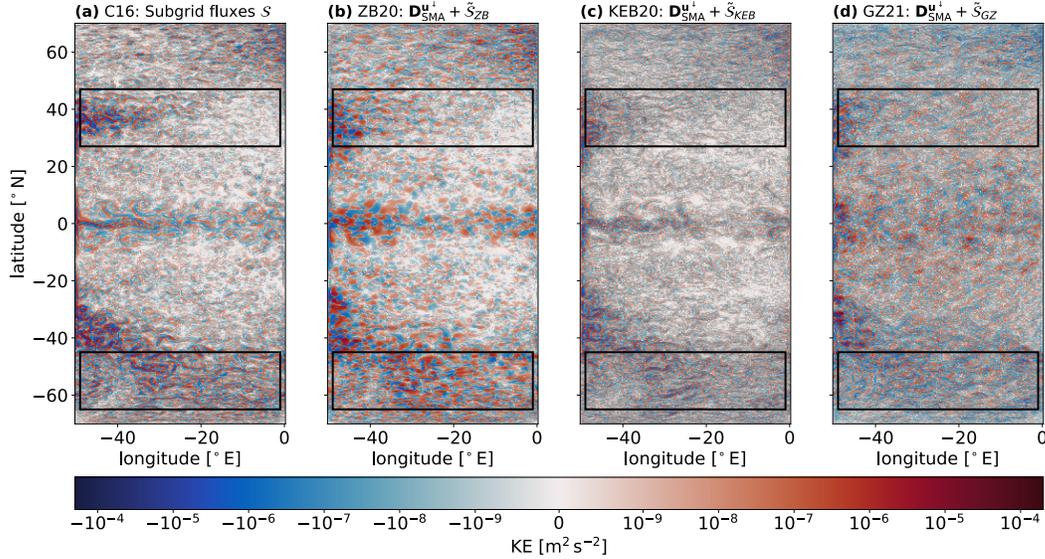
**Figure 3.** Kinetic energy integrated over depth and longitude (panel (a)) and latitude and longitude (panel (b)). The profiles correspond to averages over the last ten years of the R4 (black dotted), R16 (black solid), C16 (black dash-dotted), and the ZB20 (purple), GZ21 (blue), and KEB20 (orange) experiments. Panel (b) depicts the respective difference to the C16 ground truth for better visibility, while the inset shows the overall shape of the kinetic energy profiles, using the same color coding as panel (a).

352 The meridional KE profiles of KEB20 (orange) and ZB20 (purple) are broadly sim-  
 353 ilar. They perform well in the Southern Ocean region, with slightly less KE in the north-  
 354 ern part of the periodic channel. ZB20 barely improves the integrated KE on the equa-  
 355 tor, while KEB20 closely follows the target. Both parameterizations succeed in repre-  
 356 senting KE in the tropical bands but fail to capture the eddy-rich Gulf Stream separa-  
 357 tion region. They inject too little KE and exhibit a southward shift of the maximum KE,  
 358 as does the unparameterized reference experiment (R4). GZ21 (blue) injects too much  
 359 KE at low latitudes and too little at high latitudes. This is likely due to the non-uniform  
 360 grid size of DINO, which is roughly one-third the size at the poles compared to that at  
 361 the equator. Even though GZ21 was trained on non-uniform data, it is agnostic to the  
 362 local grid size and cannot infer spatial derivatives of the flow field (see Equation 18). This  
 363 presumably leads to poor generalization across different resolutions and dynamical regimes.

364 The depth profiles of domain integrated KE reveal a surface-intensified flow for all  
 365 experiments (inlet in panel (b) of Figure 3). Again, we find that overall, R16 is more  
 366 energetic than R4 and that, after coarse-graining, most of the additional KE is retained.  
 367 KEB20 reproduces the depth profile of the target ground truth well, with slightly higher  
 368 KE close to the surface. Both ZB20 and GZ21 underestimate KE close to the surface  
 369 and overestimate it at depth. This bias is more pronounced for GZ21, which was trained  
 370 only on surface data.

371 The results presented so far reveal only the net effect of resolved and parameter-  
 372 ized subgrid fluxes on mean KE: an upscale flux of momentum, leading to a net increase  
 373 in KE across the domain. Snapshots of surface KE tendency due to the subgrid terms  
 374 show that the associated KE transfers across the coarse grid scale can be locally upscale  
 375 or downscale, fluctuating across a wide range of spatial scales (see Figure 4).

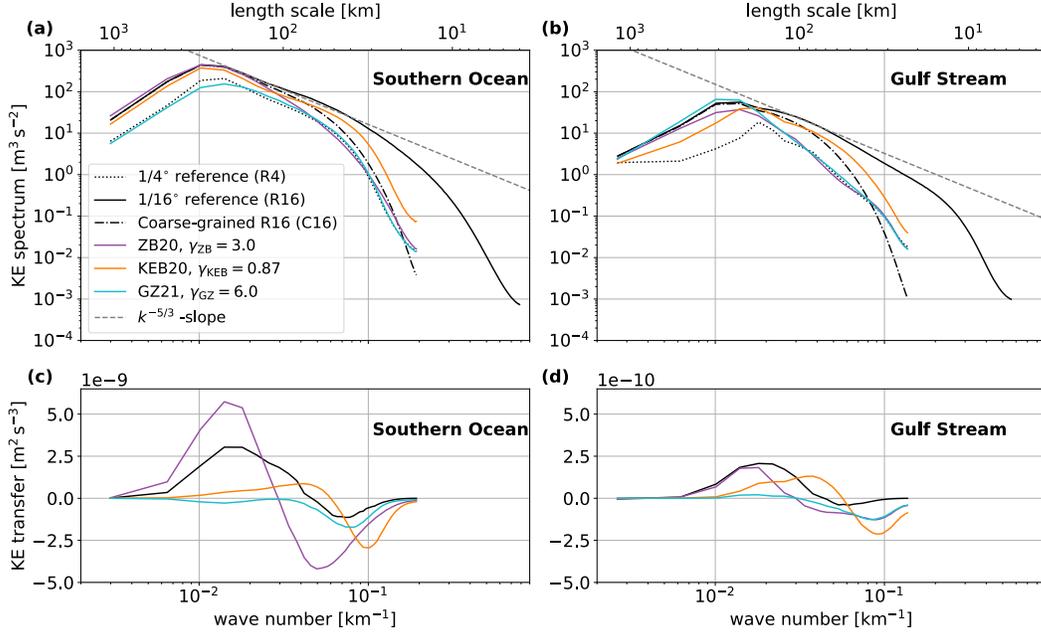
376 As already suggested by the meridional profiles of mean KE (panel (a) of Figure 3),  
 377 the most intense instantaneous fluxes are indeed found in the Southern Ocean, around



**Figure 4.** Snapshots of surface kinetic energy tendency due to (a) the diagnosed subgrid fluxes  $\mathcal{S}$  and the combined parameterized fluxes  $\mathbf{D}_{\text{SMA}}^{\text{u}} + \tilde{\mathcal{S}}$  of (b) ZB20, (c) KEB20 and (d) GZ21, as defined in Equation 8 and Equation 9. Black boxes indicate the Southern Ocean and the Gulf Stream region.

378 the Equator, and downstream of the Gulf Stream separation for all experiments. In the  
 379 case of GZ21, we also find a clear latitudinal pattern, with negative KE fluxes dominat-  
 380 ing at high latitudes, while positive KE fluxes dominate at low latitudes. A direct com-  
 381 parison of the instantaneous fields is not straightforward. Instead, we analyze the sig-  
 382 nal after projecting it into spectral space. We focus on surface fields in the two regions  
 383 indicated by the black boxes in Figure 4: the Southern Ocean (SO) and the Gulf Stream  
 384 region (GS). The fields are first interpolated to equidistant Cartesian coordinates using  
 385 the *xemf* Python package (Zhuang et al., 2023). Then, we use the *xrft* Python package  
 386 (Uchida et al., 2023) to compute the isotropic power density spectrum of KE and the  
 387 isotropic transfer spectrum from the diagnosed and parameterized KE fluxes (see Fig-  
 388 ure 5).

389 The coarse R4 experiment has less KE than the high-resolution R16 experiment  
 390 across all scales for both the SO and GS regions (panels (a) and (b) of Figure 5; note  
 391 that, overall, the GS region is approximately one order of magnitude less energetic than  
 392 the SO). The coarse-graining operator only affects higher wavenumbers, drastically reduc-  
 393 ing the KE towards the coarse grid scale. At larger scales, the increased KE is retain-  
 394 ed. This increased KE in the coarse-grained high-resolution simulation, compared  
 395 to the low-resolution simulation, demonstrates the inverse cascade, which transfers KE  
 396 from the subfilter scale all the way to the largest scales. Turbulence theory predicts a  
 397  $k^{-\frac{5}{3}}$  slope for the KE spectrum, where energy cascades from small to large scales (Kraichnan,  
 398 1967) (see the gray dashed line in Figure 5). We find that the high-resolution spectrum  
 399 follows the predicted slope between scales of 40–200 km in the SO and 50–120 km in  
 400 the GS region. We diagnose the transfer spectrum with isotropic cross-spectra between  
 401 the coarse-grained velocity field  $\mathbf{u}_h^{\uparrow}$  and the subgrid fluxes  $\mathcal{S}$  (see panels (c) and (d) of  
 402 Figure 5). On average, the subgrid fluxes extract KE at scales smaller than approximately  
 403 70 km in the SO and 80 km in the GS region, while injecting KE at larger scales. This  
 404 aligns with the range of scales predicted by the  $k^{-\frac{5}{3}}$  power law fitted to the R16 spec-  
 405 trum. Consistent with the power density spectra, the transfers in the GS region are ap-



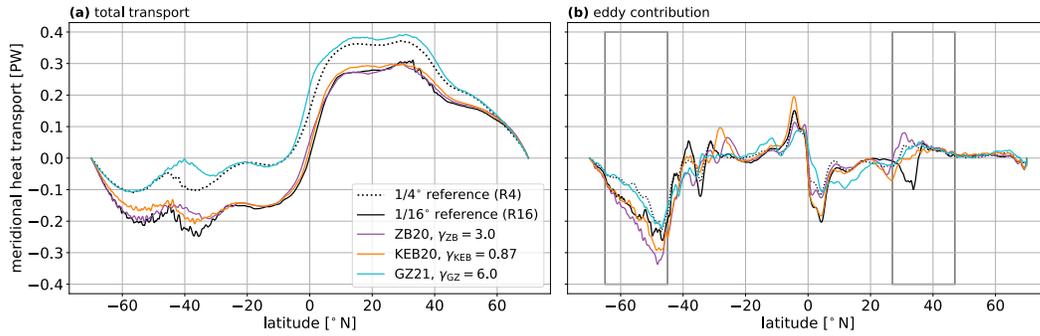
**Figure 5.** The two upper panels show the power spectrum of eddy-kinetic-energy for all experiments in the (a) Southern Ocean and the (b) Gulf-Stream region. The gray dashed line represents a  $k^{-5/3}$  power law. Panels (c) and (d) show the corresponding kinetic energy transfers due to the diagnosed (black solid line) and parameterized subgrid fluxes. They include the dissipative fluxes of the biharmonic Smagorinsky viscosity. All spectra are derived from velocity fields within the black boxes in Figure 4, interpolated onto an equidistant Cartesian grid. We use the Python library *xrft* (Uchida et al., 2023) and apply linear detrending along with a Hann window for smoothing. The spectra are averaged over the years 20 to 30 of each simulation.

406 proximately one order of magnitude smaller than those in the SO. It is these momen-  
 407 tum fluxes that are approximated by parameterizations such as KEB20, ZB20, and GZ21.  
 408 When comparing their KE transfers to the ground truth, we must also take the dissi-  
 409 pative fluxes of the Smagorinsky scheme into account (see section 2.1).

410 Overall, we find that the combined KE fluxes from parameterizations, including  
 411 contributions from the Smagorinsky scheme (Equation 9), are more dissipative than those  
 412 diagnosed from the coarse-grained ground truth (Equation 8). This demonstrates the  
 413 additional dissipation necessary for stable simulations at coarse resolution, as discussed  
 414 in Section 2.1. The positive KE fluxes of the KEB20 experiment are reduced and shifted  
 415 towards smaller scales in both the GS region and the SO. The resulting power spectrum  
 416 is less energetic for larger scales and more energetic for smaller scales compared to the  
 417 C16 target spectrum. In the GS region, these differences are more pronounced. ZB20,  
 418 on the other hand, has larger upscale and downscale fluxes that are shifted towards larger  
 419 scales in the SO. They remain shifted towards larger scales but are much reduced in the  
 420 GS region. The KE spectra in both regions suggest that ZB20 hardly affects scales be-  
 421 low 90 km, as they closely follow the coarse R4 experiment. This is due to the Gaussian  
 422 kernel  $\mathcal{G}$  (Equation 15) filtering the smaller scales of the subgrid forcing  $\mathcal{S}_{ZB}$ . GZ21 and  
 423 Smagorinsky combined have a net negative KE transfer across all scales in the SO. In  
 424 the GS region, the fluxes are dissipative towards smaller scales and only marginally posi-  
 425 tive for larger scales. The KE spectra hardly change in the SO compared to the R4 refer-  
 426 ence experiment, suggesting that there is little contribution from the parameterization

427 in that region. Scales above approximately 200km agree well with the ground truth spec-  
 428 trum of the GS region.

429 To further illustrate the different performances of the parameterizations in the SO  
 430 and GS regimes, we need to go beyond the instantaneous transfers of their predicted sub-  
 431 grid fluxes and towards the evolution of the circulation pattern over time. Even after the  
 432 relatively short integration period of 30 years, we identify substantial differences in the  
 large-scale circulation, notably the meridional heat transport (see Figure 6). The total



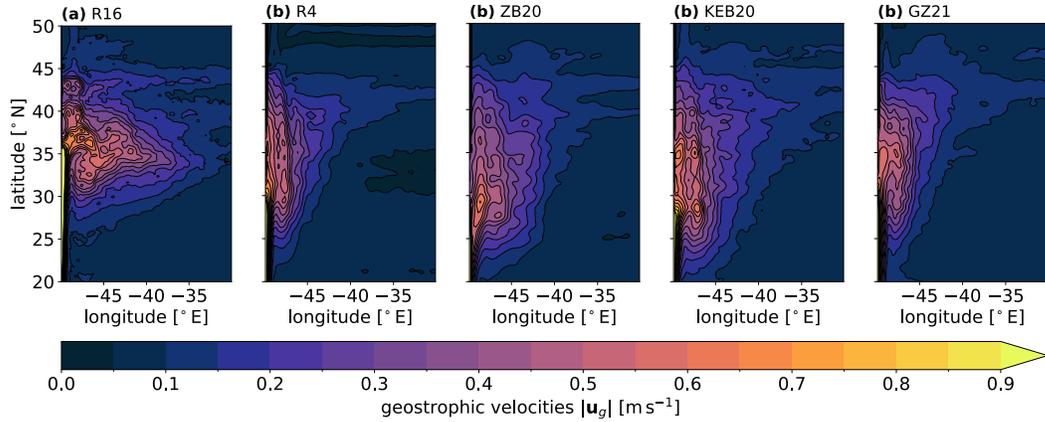
**Figure 6.** Zonally and vertically averaged meridional heat transport for all experiments. Panel (a) shows the total transport averaged over years 20 to 30 of each experiment. Panel (b) only shows the eddy contribution to the meridional heat transport. The boxes mark the latitudes of the Southern Ocean region (left) and Gulf Stream separation region (right).

433 meridional heat transport (panel (a) of Figure 6) has a stronger southward component  
 434 at higher resolution, with enhanced southward and decreased northward transport. GZ21  
 435 only has a marginal but contrary effect on the total heat transport. Both the ZB20 and  
 436 KEB20 experiments agree well with the ground truth experiment. Following previous  
 437 studies (e.g., Jayne & Marotzke, 2002), we diagnose the time mean eddy contribution  
 438 of the meridional heat transport as  
 439

$$440 \quad \langle v' \theta' \rangle = \langle v \theta \rangle - \langle v \rangle \langle \theta \rangle, \quad (19)$$

441 where we use angle brackets to denote a time average over the ten years of data collec-  
 442 tion and  $\theta$  is the potential temperature. The largest differences in the eddy contribution  
 443 in R4 versus R16 arise in the SO and GS (marked by boxes in panel (b) of Figure 6).  
 444 In the SO, R16 has an enhanced southward eddy heat transport. This is well captured  
 445 by ZB20 and KEB20, with slightly overestimated eddy transports. As for the total heat  
 446 transport, GZ21 does not reproduce this behavior.

447 After detaching from the western boundary, the Gulf Stream representation in R16  
 448 propagates almost entirely zonally eastward and leads to eddy-induced stirring against  
 449 the meridional temperature gradient. This produces a net southward eddy heat trans-  
 450 port. Some eddies are advected northward, causing a net northward eddy transport north  
 451 of the separation point. In the R4 experiment, this signal is shifted southward with re-  
 452 duced eddy-induced southward transport and increased northward transport. While ZB20  
 453 and KEB20 increase the respective northward and southward transports, none of the pa-  
 454 rameterizations improve the latitude of the Gulf Stream separation with respect to R16.  
 455 We visualize these differences in the Gulf Stream pathway with geostrophic currents de-  
 456 rived from sea surface height fields of the western part of the GS region (see Figure 7).  
 457 In the R16 experiment, we observe an intense jet in the narrow western boundary layer,  
 458 which detaches around  $36^\circ N$  and splits into several branches to the north, east, and north-  
 459 east. For all coarse experiments, with and without parameterization, the Gulf Stream



**Figure 7.** Geostrophic velocities averaged over years 20 to 30 for the (a) R16, (b) R4, (c) ZB20, (d) KEB20 and (e) GZ21 experiments. The meridional extent corresponds to the Gulf Stream separation region defined by the upper black box in Figure 4.

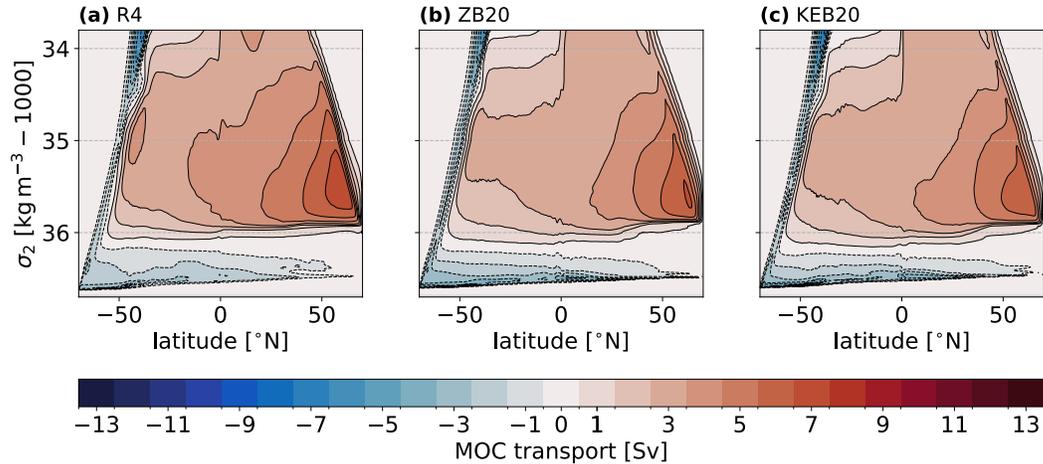
460 is much weaker, broader, and does not clearly separate from the coast, while the largest  
 461 northward velocities do not reach as far north as in R16. The sensitivity of Gulf Stream  
 462 separation to horizontal resolution, as well as the impact of topography, has already been  
 463 discussed at length (e.g., Chassignet & Xu, 2017). This mechanism is not accounted for  
 464 in the closure problem formulated here and is therefore not addressed by the subgrid fluxes  
 465 predicted by ZB20, KEB20, or GZ21.

## 466 4.2 Long-term metrics

467 We identified differences between two data-driven eddy parameterizations and a  
 468 more traditional process-based backscatter parameterization concerning their predicted  
 469 momentum fluxes. In order to assess these with respect to a high-resolution ground truth,  
 470 we are limited by computational cost to 30 years of simulation rollouts; this duration is  
 471 too short a timescale for long-adjusting climate metrics. As a compromise, we afford 600-  
 472 year rollouts of the low-resolution experiments, namely R4, ZB20, and KEB20. We ex-  
 473 clude GZ21 from this comparison due to its large biases in KE transfers, KE profiles, and  
 474 meridional heat transport, which we observe within the first 30 years. We analyze the  
 475 last 50 years of each 600-year-long experiment to compare the mean state with respect  
 476 to the slowly adjusting stratification and the associated large-scale ocean circulation.

477 Starting with the MOC in density space, we find that all experiments exhibit an  
 478 interhemispheric clockwise overturning cell above a denser anti-clockwise bottom cell (Fig-  
 479 ure 8). The upper cell is associated with the model equivalent of North Atlantic Deep  
 480 Water (NADW), forming at the surface, close to the northern boundary. There, it sinks,  
 481 flows along a nearly adiabatic pathway crossing almost the entire basin, and resurfaces  
 482 in the Southern Ocean channel. The lower cell is associated with the model equivalent  
 483 of Antarctic Bottom Water (AABW). This is the densest water mass, which forms close  
 484 to the southern boundary and flows northward along the bottom topography. It slowly  
 485 loses buoyancy through bottom mixing and returns southward, resurfacing in the south-  
 486 ern part of the periodic channel.

487 Based on the circulation pattern, we roughly define the model representation of AABW  
 488 and NADW as water with densities  $\sigma_2 > 36 \text{ kg m}^{-3}$  and  $36 \text{ kg m}^{-3} \geq \sigma_2 > 35 \text{ kg m}^{-3}$ ,  
 489 respectively. This definition does not represent the observed water mass properties of



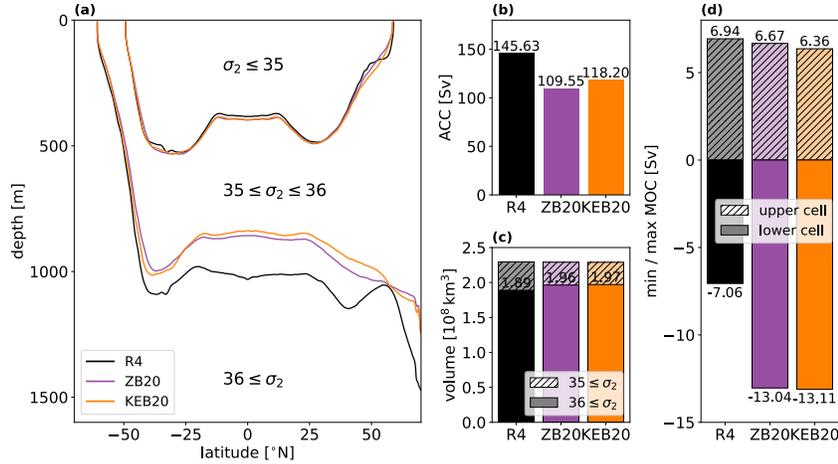
**Figure 8.** Meridional overturning circulation strength in density space for the (a) unparameterized reference experiment, (b) Zanna-Bolton 2020 and (c) Kinetic-Energy-Backscatter 2020 parameterization.

490 AABW and NADW but serves to characterize the idealized mean stratification and cir-  
 491 culation, following the model of Vallis (2017).

492 The overall overturning structure is not affected by the parameterizations; how-  
 493 ever, both ZB20 and KEB20 weaken the upper cell and strengthen the lower cell of the  
 494 meridional overturning circulation compared to the reference, with KEB20 having a stronger  
 495 effect than ZB20 (Figure 8 and panel (d) of Figure 9). As we cannot afford to run the  
 496 ground truth experiment for the same adjustment time, we cannot comment on which  
 497 is more realistic. These changes in the large-scale circulation are not a direct consequence  
 498 of the KE transfers discussed in the previous section, but arise indirectly through slow  
 499 changes in stratification caused by the intensified eddy field. Indeed, more energetic ed-  
 500 dies lead to enhanced lateral stirring of tracers, ultimately flattening the isopycnal sur-  
 501 faces. We find this effect to be most pronounced in the Southern Ocean channel, where  
 502 the isopycnals are steeper and backscatter is stronger (as shown for the  $\sigma_2 = 27 \text{ kg m}^{-3}$   
 503 and  $\sigma_2 = 26 \text{ kg m}^{-3}$  isopycnals in panel (a) of Figure 9). This leads to substantial changes  
 504 in stratification: the volume of AABW increases at the expense of NADW for both KEB20  
 505 and ZB20 (see panel (c) of Figure 9). Changes in the volumes of AABW and NADW  
 506 are ultimately reflected in the overturning circulation strength: more AABW is associ-  
 507 ated with a stronger lower cell, and less NADW with a weaker upper cell (panel (d)  
 508 of Figure 9). The changes in stratification also impact the ACC through thermal wind bal-  
 509 ance: the steeper the slope of the isopycnals in the channel, the stronger the current. Con-  
 510 sequently, we diagnose the strongest transport in the unparameterized reference exper-  
 511 iment (see panel (b) of Figure 9). ZB20 exhibits the flattest isopycnals overall and, ac-  
 512 cordingly, the weakest ACC transport. The presented response patterns of the ACC, MOC,  
 513 and stratification are directly relevant for the application of backscatter parameteriza-  
 514 tions in global ocean models (e.g., Juricke et al., 2020; Grooms et al., 2025; Perezhugin  
 515 et al., 2025).

## 516 5 Discussion

517 The assessment of two data-driven eddy closures performed in this study raises sev-  
 518 eral issues regarding their a priori training methodology and generalizability to differ-



**Figure 9.** Comparison of (a) zonal mean stratification, (b) ACC strength, (c) total volumes of NADW (hatched) and AABW (non-hatched) and (d) the maximum overturning strength of the lower (non-hatched) and upper (hatched) cell for the R4, ZB20 and KEB20 experiment.

ent ocean models and dynamical regimes. In this section, we will discuss these findings and propose possible avenues for addressing them in future studies.

We note that the coarse-graining approach employed in both studies fails to yield a numerically stable model at runtime. This is expressed by the difference between the governing equations of the coarse grained high resolution (Equation 6) and the low resolution (Equation 1) experiment, where the latter requires a viscous dissipation term  $\mathbf{D}_{\text{SMA}}^{\text{u}}$  to suppress gradients near the grid scale. The inconsistency between the coarse-grained target model and the coarse model used for simulation rollouts is a known issue, causing poor a posteriori (online) performance of closures that were trained a priori (offline) (Sanderson et al., 2024). A promising approach to this problem is online learning, where the parameterization is trained during runtime, adjusting the model trajectory step by step. This method implicitly informs the ML model about the coarse model, which has been found to lead to more stable online simulations (see e.g., Rasp, 2020; Frezat et al., 2022). However, this is not straightforward for general circulation models like NEMO, as it requires a differentiable solver or a differentiable adjoint model (Frezat et al., 2024). An alternative approach for this specific case could be to inform the ML model about the additional dissipative fluxes explicitly through the Smagorinsky scheme, since we can diagnose them a priori from the coarse-grained data (Mana & Zanna, 2014). One approach could be to follow the considerations from more traditional backscatter closures, such as KEB20, to constrain the domain-integrated upscale KE transfer of the predicted fluxes  $\tilde{\mathcal{S}}$  by the domain-integrated downscale fluxes of the viscosity operator during training. While there have been efforts for the combined treatment of the dissipative and backscatter fluxes (e.g., Perezhogin & Glazunov, 2023), none of these approaches eliminates the need for additional numerical dissipation in coarse-resolution ocean models and the associated risk of error compensation. This remains an open question for the ocean modeling community.

As ZB20 and GZ21 were already trained offline prior to this evaluation study, we employed an online tuning strategy targeting the total integrated KE, thereby allowing for a controlled compensation of the numerically imposed dissipation and backscatter during the evaluation. The tuning procedure exposed an explicit dependency of the sub-grid fluxes on the size of the grid cell, which was not detected during the training phase. For ZB20, the grid dependency is accounted for through the discretization of the discov-

ered equation and the association of its free parameter to the local grid scale (see Equation 17) following the theoretical arguments of previous studies. A key advantage of the ZB20 approach thus lies in its interpretability, in contrast to the black-box deep learning approach of GZ21, which is agnostic to the model grid. ZB20 outperforms GZ21 across all metrics shown in this study, suggesting that information about the grid is essential for the generalizability of ML-based eddy parameterizations (see Perezhugin et al., 2025, for a detailed discussion).

We did not tune each vertical layer independently, nor with horizontally varying coefficients, as suggested by Zhang et al. (2023). This decision was motivated by two factors: First, this would involve too many simulation rollouts for the a posteriori approach chosen here. Second, it could lead to overfitting: in a first attempt for GZ21, we found that the optimal vertical profile of  $\gamma_{GZ}(z)$  would invert the sign of the predicted net KE flux for a few layers at the bottom of the mixed layer. While this demonstrates the difficulty for GZ21 to generalize from surface training data to the baroclinic flow of the model, injecting energy where the model predicts extraction and vice versa suggests overfitting rather than capturing a physical mechanism. Similarly, dependencies on the grid, latitude, or flow should be detected in the training phase of the data-driven approaches.

The KE and KE transfer spectra (see Figure 5) show that ZB20 is more effective in energizing the large-scale than the baseline backscatter parameterization KEB20. The scales at which energy is injected in KEB20 arise from the negative Laplacian operator, which is not strictly derived from first principles. These differences in the instantaneous KE transfers only translate to minor differences in the slowly adjusting mean state. Both parameterizations energize the eddy field, leading to enhanced tracer stirring and a flattening of isopycnal slopes, especially in the Southern Ocean. This modifies the water mass properties at depth, slowly changing the large scale circulation towards a weaker ACC and upper MOC cell and a stronger bottom MOC cell. The described mechanism is consistent with the model study by Marshall et al. (2017) and the backscatter parameterization experiments performed in a global ocean model by Juricke et al. (2020). We find that ZB20 is slightly more effective in flattening the isopycnals compared to KEB, resulting in a weaker ACC. KEB20, on the other hand, produces more AABW at the expense of NADW, favoring a slightly stronger deep cell and a slightly weaker upper cell of the MOC.

Ultimately, we cannot evaluate these long-term metrics quantitatively with respect to the high-resolution ground truth, as we cannot afford 600 years of the R16 experiment. Nevertheless, the spectra, KE profiles, and integrated meridional heat transport of the first 30 years indicate that ZB20 and KEB20 both perform well in the Southern Ocean, but not in the Gulf Stream region. We attribute this to a misrepresentation of the bathymetry and the western boundary layer at coarse resolution. Previous studies in global ocean models found that backscatter improves the Gulf Stream separation, mainly by countering the excessive numerical dissipation of the western boundary current at coarse resolution (Juricke et al., 2020; Chang et al., 2023). We cannot reproduce these results in the idealized model used in this study. At coarse resolution, the Gulf Stream is overall less energetic, and none of the tested backscatter parameterizations lead to a clear improvement in the separation or path of the current. The DINO configuration has a very idealized bathymetry, with an exponential slope at the boundary and vertical walls above 2000 m (see Appendix A of Kamm et al., 2025, for details). Western boundary currents can, therefore, only be represented in a simplified way. In this context, the representation of bathymetry affected by horizontal resolution may play a more significant role than KE pathways in controlling the separation and course of the Gulf Stream. This is not addressed by any of the parameterizations presented here and does not emerge naturally from the coarse-graining method of ZB20 and GZ21. There are other approaches to this problem, such as terrain-following representations of the bathymetry or volume penal-

603 ization informed by the high-resolution bathymetry (Debreu et al., 2022), but these are  
 604 beyond the scope of this study.

## 605 **6 Conclusion**

606 We have implemented and tested two data-driven eddy parameterizations, Zanna  
 607 and Bolton (2020) (ZB20) and Guillaumin and Zanna (2021) (GZ21), in the NEMO ocean  
 608 model, using a configuration of intermediate complexity (Kamm et al., 2025). Our work  
 609 is pioneering, as it is among the first to integrate a neural network into the NEMO model.  
 610 Both parameterizations were trained a priori on subgrid fluxes diagnosed from coarse-  
 611 grained, high-resolution data generated by different ocean models than the one in which  
 612 they were tested. We emphasize that this approach neglects the enhanced viscous fluxes  
 613 necessary for numerical stability in coarse simulation rollouts. We compensate for this  
 614 by tuning the combined fluxes of each parameterization and the viscosity scheme a pos-  
 615 teriori to match the domain-integrated kinetic energy of the ground truth.

616 GZ21 does not generalize well to the previously unseen model data. It was trained  
 617 only on surface data and is agnostic to the spatial grid. The closure, therefore, tends to  
 618 underestimate the KE backscatter at high latitudes and near the surface while overes-  
 619 timating it at low latitudes and at depth. It does not improve the KE spectra or the large-  
 620 scale circulation patterns compared to the unparameterized control run. ZB20 general-  
 621 izes better, as it is inherently grid-aware through the discretization of the discovered equa-  
 622 tion and the dimensional scaling introduced by Perezhogin et al. (2024). The KE spec-  
 623 tra resemble the ground truth well, especially in the Southern Ocean. ZB20 exhibits KE  
 624 spectra closer to the R16 experiment when compared to the KEB20 baseline experiment,  
 625 since it energizes the large scale more efficiently. Both ZB20 and KEB20 improve the basin-  
 626 scale circulation through the energized eddy field. In the Southern Ocean, the enhanced  
 627 tracer stirring flattens the isopycnals, leading to a weaker ACC, a stronger deep cell of  
 628 the MOC, and a weaker upper cell of the MOC, which is in line with previous studies  
 629 (Marshall et al., 2017; Juricke et al., 2020).

630 We cannot assess whether the data-driven ZB20 parameterization outperforms the  
 631 baseline in terms of its effect on the mean state, as we cannot afford to run the ground  
 632 truth to thermodynamic equilibrium. Based on the 30 years available, we nevertheless  
 633 identify substantial differences in the mean flow pattern in the Gulf Stream region. All  
 634 parameterizations evaluated here produce a Gulf Stream that is too weak, overly broad,  
 635 and separates too far south. We attribute this to the misrepresentation of the idealized  
 636 bathymetry at coarse resolution, which is not addressed by the backscatter parameter-  
 637 izations presented in this study.

638 The study offers insight into the impact of the training strategy for data-driven eddy  
 639 parameterizations, particularly regarding their generalizability across different model-  
 640 ing environments. As guidance for future developments, we propose to *i)* inform data-  
 641 driven parameterizations with the grid to allow for the inference of spatial derivatives  
 642 and *ii)* consider all resolution-dependent model components when formulating the tar-  
 643 get ground truth. For *ii)*, we highlight the excessive numerical dissipation introduced  
 644 by the viscosity scheme at coarse resolution, which leads to a discrepancy between the  
 645 a priori and a posteriori behavior of the data-driven parameterizations. Finally, we ad-  
 646 vocate that the ultimate aim of the data-driven approach should be to develop physi-  
 647 cally interpretable momentum closures, as this facilitates generalizability to other mod-  
 648 eling frameworks.

## Data Availability Statement

The DINO configuration is available via Kamm et al. (2025), and all experiments presented here can be reproduced using Kamm (2026b). The processed simulation data and the code to generate all figures in this manuscript are available via Kamm (2026a).

## Conflict of Interest declaration

The authors declare there are no conflicts of interest for this manuscript.

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